Exam Seat No:________________ Enrollment No:__________________

C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013

Course Name :B.ScSemester-I **Subject Name: -Mathematics -I**

Duration :- 2:30 Hours **Date : 02/12/2013**

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places.
(5) Assume suitable & Perfect data if needed

SECTION-I

Q1 a) State Taylor's theorem. 2

b) If $y = e^x$, then $y_{16} =$ ______. 1

c) Center of the sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$ is _____. 1

d) If $y = \cos(ax + b)$, then find y_n. 1

e) Transform $\theta = 30^0$ in Cartesian form. 1

f) Prove that $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n.$ 1

Q2 a) Find a, b, c so that $\lim_{x \to 0} \frac{a\,e^x - 2\,b\cos x + 3\,c\,e^{-x}}{x \sin x} = 2.$ 5

b) State and prove Leibnitz's theorem. 5

c) Find the Maclaurin's expansion $f(x) = \sin x$. 4

OR

Q2 a) Evaluate: $\lim_{x \to 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$. 5

b) Let $y = (x^2 - 2)^m$. Find the value of m such that $(x^2 - 2)y_{n+2} + 2\,x\,y_{n+1} - n(n+1)y_n = 0.$ 5

c) Find y_n for $y = e^{2x} \cos x \sin 2x$. 4

Q3 a) State and prove Lagrange's mean value theorem. 6

b) If any straight line through the pole meets the circle $r^2 - 2rd\cos(\theta - \alpha) + d^2 - a^2 = 0$ at point P and Q. Then prove that $OP \cdot OQ = d^2 - a^2$. 4

c) Show that following pair of spheres touch each other 4
$x^2 + y^2 + z^2 = 64,\qquad x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0.$

OR

Q3 a) Let two spheres be given by 6
$S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0, S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 =$
Then prove that $S_1 + \lambda S_2 = 0$, where $\lambda \in R, \lambda \neq -1$, represents a family of spheres passing through the intersection of the spheres $S_1 = 0$ and $S_2 = 0$.

b) In usual notation prove that polar equation of circle is 4
$r^2 + r_1^2 - 2rr_1\cos(\theta - \theta_1) = a^2.$

c) Verify the Roll's theorem for the function 4
$f(x) = x^2 - 2x + 3, x \in [0, \quad 2].$

SECTION-II

Q4 a) What is the order and degree of $\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} + xy = x^3$. 2

b) Define symmetric matrix. 1

c) Define scalar matrix. 1

d) Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ in reduced row echelon form? 1

e) Is the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ in reduced row echelon form? 1

f) Solve $x\,dx - y^2\,dy = 0$. 1

Q5 a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. 5

b) Solve the system $3x + 2y - z = 4, \quad x + 6y + 3z = 22, \quad 2x - 4y = -6$. 5

c) Find rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$. 4

OR

Q5 a) Find normal form of the matrix $A = \begin{bmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 11 & 12 & 13 & 14 \end{bmatrix}$ and hence rank of A . 5

b) Find inverse by Gauss Jordan Method, for $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$. 5

c) Solve the system by Gauss Jordan method 4

$3x - y - z = 0, \quad x + y + 2z = 0, \quad 5x + y + 3z = 0.$

Q6 a) Find eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. 6

b) Solve: $x^2y\,dx - (x^3 + y^3)dy = 0$. 4

c) Solve: $\frac{dy}{dx} + y\cot x = 4x\,cosec\,x$ 4

OR

Q6 a) Verify Cayley‒ Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 6

b) Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$. 4

c) Solve: $(p + y + x)(p + 2x) = 0$, where $p = \frac{dy}{dx}$. 4

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